

# POTENTIAL HONOURS PROJECTS AND SUPERVISORS

2008

## Department of Mathematical Sciences

The Department has an active research culture and are involved with three of UTS's research strengths: CUDOS, CenSoC and QFRC. Honours students get a chance to be involved in research work in one of these centres.

### CUDOS

The UTS Node of CUDOS focuses on theoretical and computational modelling of photonic devices and undertakes both fundamental and applied research in pursuit of the centre's goal of developing an all optical signal processor, also known to as the "photonic chip". Our work at UTS concerns the modelling of propagation in photonic devices and the means by which they influence emission from sources (e.g., micro lasers) embedded within them. Much of our work is related to devices based on photonic crystals—periodic structures which are the optical analogues of semiconductors, possessing frequency bands in which propagation is allowed and other bands, called photonic band gaps, in which propagation is partially inhibited or not possible. In this way light the flow of light can be controlled and the radiation properties of sources tailored by the geometry of the structure. While photonic band gaps underpin the technological potential of photonic crystal structures, it is the introduction of defects into the otherwise periodic structure that allows this potential to be realised. Examples of defect structures are waveguides, resonators, filters, photonic crystal fibres and many more.

The rigorous modelling of such devices involves the solution of the Maxwell's equations of electromagnetism using analytic and numerical approaches. In general, the structures that are of contemporary interest are too complex to be handled by exclusively analytic approaches and so some aspect of numerical simulation is required, with such methods generally referred to as semi-analytic. In some cases, the structure is so general, or so complex, that purely computational approaches are required, with finite difference and finite element methods being the most common. In many cases, one is interested in the steady state solution of the field problem and so here one general chooses to work in the frequency domain. In other cases, however, there is a need to obtain a time resolved solution and here the finite difference time domain (FDTD) algorithm is the most widely adopted approach. In the case of FDTD calculations, we undertake our work on desktop systems (for small problems) and on supercomputing facilities at ac3 in Sydney and the APAC National Facility in Canberra.

While these purely numerical methods provide quite accurate results, they do not provide much in the way of physical insight into the scattering processes involved, and nor do they take advantage of the geometrical structure to enhance the computational efficiency. It is here that semi-analytic methods may come to the fore, where the structure is amenable to this form of treatment, with these taking advantage of the periodicity to express fields in terms of eigenfunctions (known as Bloch modes), thereby providing outstanding accuracy, high computational efficiency, analytic tractability (e.g., allowing the derivation of elegant asymptotic expansions of certain quantities) and excellent physical insight.

With this as a general background, we would be happy to discuss the details of any of the projects below with potential honours students.

**Topic:** Impedance of photonic crystals

**Supervisors:** Lindsay Botten, Kokou Dossou and Ara Asatryan

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**Description:** Photonic crystals are structures in which the refractive index varies periodically with position. They therefore have (photonic) band structure with a (photonic) bandgap. While many of the applications rely on the presence of bandgap, many others, for example propagation without diffraction, do not, and require particular features in the bands. Such applications require the reflectivity of the structure to be minimized since otherwise the reflected light may end up where it becomes a nuisance. Now, in uniform media the reflectivity is calculated using impedances, and we have recently succeeded in defining impedances for photonic crystals as well, but only in the simplest case. Nonetheless, this allowed us to devise an anti-reflection coating for photonic crystals, which is virtually impossible to do by any other method. The aim of this project is to generalize the concept of impedance to more complex structures, and then to demonstrate its utility by making an otherwise very difficult problem do-able.

**Topic:** Suppression of the localisation of light in metamaterial structures

**Supervisors:** Ara Asatryan and Lindsay Botten

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**Description:** One of the most fascinating and universal phenomena in physics is localization, whereby the propagation of waves is suppressed by the presence of disorder. While localization has been studied for many years there has been considerable recent interest in metamaterials, materials in which the group and phase velocities point in opposite directions. We recently undertook the first study combining conventional materials and metamaterials and, to our surprise, found that metamaterials can substantially suppress the localisation of light. Not only is the decay rate orders of magnitude larger at long wavelengths, it is also quantitatively different (it scales with the sixth power of wavelength rather than the second). The aim of this project is to understand this phenomenon better by investigating, theoretically and computationally, its dependence on the position of the metamaterial and normal layers, and the influence of disorder in the layers' thickness.

**Topic:** Slow-light induced optical forces near photonic crystals

**Supervisors:** Adel Rahmani, Ara Asatryan, Chris Poulton, Mike Steel (RSoft) and Lindsay Botten

**Contact:** adel.rahmani@uts.edu.au

**Description:** For several decades now, physicists and biologists have used light beams to manipulate small clumps of matter through optical forces. In fact, with optical devices that can confine light to a very small domain, such as photonic crystals, it is possible to enhance the optical force up to the point where it can bend, rotate or displace a small object placed nearby or the device itself. Such forces are invaluable for moving and sorting interesting molecules like DNA, or for exploring new regimes in the physics of the coupling between light and mechanical resonances in small objects. The aim of this project is to explore the physics of optical forces in the context of photonic crystals (PC). Using semi-analytic and advanced computational methods on supercomputers, we will investigate how the special properties of PC slow light modes can be used to enhance the strength and flexibility of optical forces.

**Topic:** Scattering in Photonic Crystal Fibres

**Supervisors:** Chris Poulton, Adel Rahmani and Lindsay Botten

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**Description:** Photonic Crystal Fibres (PCFs) are devices with a range of important applications in physics and telecommunications. They function by changing the ability of light to propagate out of the centre of the fibre, thereby forcing the light to be confined within a small core. This core may be hollow, and researchers are now beginning to examine the effects of putting small particles in the core of the fibre, with a view to constructing novel nanophotonic devices such as filters and resonators. When light is scattered by the particle, the particle will experience radiation pressure, causing it to move in a complicated way which, in turn, will change the amount of scattered light. The aim of this project is to study the physics of these complicated light-matter interactions, using a combination of theoretical analysis and high-performance computing.

**Topic:** Modelling the interaction between photonic crystal cavity and waveguide

**Supervisors:** Kokou Dossou and Lindsay Botten

**Contact:** kokou.dossou@uts.edu.au

**Description:** Photonic crystals (PC) are periodic dielectric structures that have a range of interesting and useful properties. While band gaps caused by Bragg scattering provide the mechanism by which light is confined within a PC, it is the introduction of defects (e.g., waveguides, cavities, etc.) into structures which are otherwise periodic that allows useful devices to be fabricated. For instance, the introduction of a resonant cavity adjacent to a waveguide leads to a compact and efficient filtering and switching device. The aim of this project is to undertake a systematic theoretical and computational simulation of the waveguide-cavity systems. This will be undertaken using a range of theoretical and computational tools (based on the Bloch mode methods) developed by our group, and also a recently developed asymptotic theory with which we may model the cavity defect mode.

## Constructions of Optimal Designs

The Centre for the Study of Choice (CenSoC) was founded in early 2003 as a joint initiative of the School of Marketing in the UTS Faculty of Business and the Department of Mathematical Sciences in the UTS Faculty of Science. CenSoC is codirected by Professor Michael Keane, an Australian Federation Fellow working in labour economics and the modelling of choices of individuals and organisations and located at UTS, Professor Jordan Louviere in the School of Marketing and Professor Deborah Street in the Department of Mathematical Sciences (Statistics).

The primary objective of CenSoC is to invest in research and development that leads to better understanding and prediction of individual and group decisions, including the choice processes of managers, organisations and consumers.

Understanding and predicting decisions/choices matters academically, commercially and societally because decisions made by individual consumers, voters, governments and firms impact not only their immediate areas of interest, but also wider communities. As well, many decisions and choices made by individuals and groups drive everyday aggregate market and community outcomes like aggregate movements in stock market indices, supply and demand for goods and services, electoral outcomes, and many more.

In the Department of Mathematical Sciences our work for CenSoC focuses on the development of new theoretical constructions for optimal discrete choice experiments. We are also actively engaged in designing experiments for the Centre for Health Economics Research Evaluation (CHERE), the ATN Centre for Metabolic Fitness and other academic and commercial groups.

Debbie Street works in the area of the design of experiments. She has supervised a number of project students and PhD students in various aspects of the design of experiments. Some possible topics for honours projects are presented below but projects in other topics in the area can be also be supervised.

**Topic:** Eliciting Sensitive Information by Randomised Response

**Supervisor:** Prof D. Street

**Contact:** Deborah.Street@uts.edu.au

**Description:** Randomised response is a technique used in surveys to obtain estimates of the prevalence of sensitive behaviours while maintaining complete confidentiality for the survey respondents.

Balanced incomplete block designs are one way to design randomised response surveys. We have shown that blocking sets provide a useful way to ensure that no survey consists entirely of sensitive questions. But some surveys may consist entirely of innocuous questions and so not be treated seriously by respondents. This suggests (Wallis, 2003) that BIBDs in which two disjoint blocking sets exist, one for the sensitive questions and one for some innocuous questions, would be even more useful. In this project you will investigate the structure of BIBDs with two disjoint blocking sets.

**Topic:** Stated Choice Experiments

**Supervisor:** Prof D. Street

**Contact:** Deborah.Street@uts.edu.au

**Description:** In a stated choice experiment a number of respondents are each, independently, shown several choice sets, where each choice set consists of two, or more, options. Each respondent is asked to indicate their preferred option from each choice set, independently of the options in any previous choice set that they may have seen.

Stated choice experiments are widely used in many areas including marketing, transport, environmental resource economics and public welfare analysis. The optimal allocation of options to choice sets can make a substantial difference to the cost of running such an experiment.

In most applications the options are represented by the levels of 2, or more, attributes (factors). If the results of the experiment are analysed using the MNL model, the problem of the optimal allocation of options to choice sets has been solved for the estimation of main effects for any number of attributes, each having any number of levels, attributes and for the estimation of main effects and two-factor interactions when all attributes are binary. Partial results exist for other situations. In this project these partial results would be extended.

**Topic:** Designs for Group Testing

**Supervisor:** Prof D. Street

**Contact:** Deborah.Street@uts.edu.au

**Description:** Group testing is an effective way to identify a few "interesting" items in a large set of items. The items may be blood samples (find the small number of samples that test HIV positive) or DNA sequences in a DNA library (find particular DNA samples).

In the group test a number of items are pooled together and tested simultaneously. If the test is negative then no item in the pool is positive and so none of the items in that pool need to be tested further. If the test is positive then further testing is required to identify the one (or more) positive items in that pool.

Often several tests can be done in parallel and then it is sometimes possible to identify positive items from the sets in which they appeared. In this project we will look at the construction of two-dimensional "grid-block" designs for testing for DNA sequences in a DNA library.

## Quantitative Finance and Management

The Quantitative Finance Research Centre is a joint initiative of the School of Finance and Economics and the Department of Mathematical Sciences.

The Quantitative Finance Research Centre (QFRC) aims:

- To coordinate and develop a major research program in quantitative finance to address the major research problems in financial risk measurement and management.
- To provide a world class post-graduate research program in Australia, by providing a focal point for quantitative finance researchers in Australia and encouraging international research and post-doctoral student networks in the area of quantitative finance.
- To provide independent high quality policy advice to industry and government and to enhance the quality of contract research and consultancy services.

**Topic:** Estimation of financial market risk using Value at Risk and Expected Shortfall

**Supervisor:** Dr Davy Wong

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**Description:** Value at Risk (VaR) is a measure of the maximum loss of a portfolio over a predetermined horizon. More precisely, it is the loss that will be exceeded with probability  $p$  over a time horizon of  $h$  periods. Every financial institutions have to calculate their VaR associate with  $p = 1\%$  and  $h = 10$  days for determination of their market risk capital requirement. In Artzner et al (1997,1999), VaR is shown to be an incoherent risk measure. For example, if two portfolios are merged to form a combined portfolio. The VaR of the combined portfolio may exceed the sum of the VaRs of the two separate portfolios which is against the intuition of reducing risk via diversification. Acerbi and Tasche (2002) proposed a new coherent risk measures called Expected Shortfall which is the expected value of loss that exceeded VaR.

In this project, you will study the statistical properties of multiple-period returns which are the aggregates of 1-period returns under GARCH model. Then, you will construct some methods of estimating the multiple-period VaR.

Acerbi, C. and D. Tasche (2002). On the coherent of expected shortfall. *Journal of Banking and Finance* **26**, 1487-1503.

Artzner, P. Delbaen, J.M. and D. Heath (1997). Thinking coherently. *Risk* **10** (11), 68-71.

Artzner, P. Delbaen, J.M. and D. Heath (1999). Coherent measures of risk. *Mathematical Finance* **9**, 203-228.

**Topic:** Dynamic Modelling the Dividend, Investment and Financial Decisions of Firms

**Supervisor:** Dr L. Groen

**Contact:** Layna.Groen@uts.edu.au

**Description:** Slight relaxations from the assumptions of the seminal work by Modigliani and Miller result in a lack of investment and financing separation and dividend relevance. Dynamic models of the firm seek to determine the investment, debt, new equity and dividend decisions simultaneously under complex interest rate regimes. Market imperfections further complicate the situation - the firm may not have access to all investment opportunities, investors may not have access to all information. This topic will seek to examine the foundations of dynamic models of the firm using optimal control.

The topic is suitable for students who have undertaken the Bachelor of Mathematics and Finance or from the Bachelor of Science in Mathematics who have undertaken the Finance submajor.

**Topic:** Applications of Optimal Control to Product Quality Decisions (Analytical and Numerical Approaches),

**Supervisor:** Dr L. Groen

**Contact:** Layna.Groen@uts.edu.au

**Description:** If consumer response to reductions in product quality are not immediate producer firms can exploit asymmetric information, and revenue gains can result. This topic examines the dynamic relationship between a firm's decision to vary the quality of a product, consumer response to this variation and the effect this has on the firm. This is achieved through the construction and analysis of a dynamic model of the firm which incorporates a numerical model of consumer response. The consumer response model can be based on sampling the product's market. The model can be extended to include a reinstatement of quality.

The topic is suitable for students who have undertaken the Bachelor of Science in Mathematics who have majored in Operations Research.

**Topic:** Portfolio Selection with Equities and Options.

**Supervisor:** Dr L. Groen

**Contact:** Layna.Groen@uts.edu.au

**Description:** Equities are often combined with equity options in the construction of investment portfolios, yet standard portfolio optimisation techniques do not include options. In fact, static models cannot be used when options are included in portfolios. This topic will examine the formation of a portfolio of equities and options using dynamic programming applied to real data.

The topic is suitable for students who have undertaken the Bachelor of Mathematics and Finance or from the Bachelor of Science in Mathematics who have majored in Operations Research and undertaken the Finance submajor.

**Topic:** Optimisation Methods for Manpower Planning and Scheduling.

**Supervisors:** Dr Y. Zinder and Dr R. Sorli. Y. Zinder will supervise the development of optimisation algorithms. R. Sorli will supervise the computational part of the thesis.

**Contact:** Yakov.Zinder@uts.edu.au

**Description:** The project will introduced a student to the realm of optimization problems arising in various companies and organisations including supermarkets, airports, call centres, etc. The student will learn various methods, approaches and mathematical models utilised in personnel scheduling, including allocation of shifts, staff rotation, etc. The student will acquire an experience in the development of optimisation algorithms in this important field of management science.

**Topic:** Optimisation Methods in Process Scheduling.

**Supervisor:** Dr Y. Zinder

**Contact:** Yakov.Zinder@uts.edu.au

**Description:** Process scheduling is concerned with scheduling of production processes involving chemical and physical transformation of materials. Typical examples are dairy industry, brewery, and oil industry. The project will introduce a student to this relatively new field of operations research. The student will learn various optimisation techniques and will acquire an experience in the development of optimisation algorithms that are applicable in dairy industry.

**Topic:** Optimisation algorithms for deterministic scheduling models.

**Supervisor:** Dr Y. Zinder

**Contact:** Yakov.Zinder@uts.edu.au

**Description:** The project is concerned with mathematical methods for deterministic scheduling models that are applicable in computer science, manufacturing, railway scheduling, etc. The project will provide a student with an experience in the development of optimisation scheduling algorithms, and in analytical and experimental analysis of different discrete optimisation techniques.

**Topic:** Numerical Experiments in Column Generation.

**Supervisor:** Dr Phil. Neame

**Contact:** Philip.Neame@uts.edu.au

**Description:** Many large-scale optimization problems in areas such as aircrew scheduling and vehicle routing are solved by a technique known as column generation. This basically extends the revised Simplex method to cope with millions or even billions of variables. However, such problems are usually massively degenerate, meaning that there are many ways to calculate a "reduced cost", and none knows which is best. This project would focus on numerical experiments on alternative approaches, and would suit a student with programming skills.

**Topic:** Integer Variables in Portfolio Optimisation.

**Supervisor:** Dr Phil. Neame

**Contact:** Philip.Neame@uts.edu.au

**Description:** The "standard" problem in portfolio optimisation assumes that all investments are divisible (e.g. you could spend \$0.01 on an investment if that is optimal). This model is usually solved by nonlinear programming techniques, and provides a good approximation to the case for bonds and shares. However, for sizable investments in property and alternative investments (such as infrastructure projects),

the decision may be whether to invest millions of dollars, or not. This project will explore different models for these situations, most likely using mathematical modelling packages like LINGO, and would suit a BMathFin student, or a student with a Finance minor.

**Topic:** Applications of Lie Group Symmetry Methods to Option Pricing and stochastic processes.

**Supervisor:** Dr Mark Craddock

**Contact:** mark.craddock@uts.edu.au

**Description:** To find the correct price of an option one needs to solve a related Partial Differential Equation. Lie group symmetry methods provide a powerful algorithmic approach to solving many PDE's. This is suited to a BMathFin student or a BSc student who is interested in applications in Finance.

**Topic:** Numerical methods for pricing exotic options.

**Supervisor:** Dr Mark Craddock

**Contact:** mark.craddock@uts.edu.au

**Description:** This project will focus on pricing options and bonds numerically using Laplace transforms or PDE techniques. This project is suited to BSc and BmathFin students who are interested in numerical analysis.

**Topic:** Bayesian Modelling of Trends Using Geometric Process (GP)

**Supervisor:** Dr B. Choy and Dr J. Chan (University of Sydney)

**Contact:** Boris.Choy@uts.edu.au

**Description:** The GP can model different trend patterns more efficiently and give a better interpretation of the data. In this project, GP model will be employed to model the trends of financial data or insurance data. Bayesian freeware WinBUGS will be used to perform complicated Bayesian computation.

## Further Projects

**Topic:** Nonparametric regression

**Supervisor:** Dr Davy Wong

**Contact:** Chiming.Wong@uts.edu.au

**Description:** In linear regression, the regression function is assumed to be linear. However, many data sets exhibit nonlinearity. A non-linear regression function can be estimated either parametrically or non-parametrically. A parametric model assumes that the functional form of the regression function is known up to a small number of parameters. The resulting regression estimate is a curve which is selected from the family of curves allowed under the model and conforms to the data in some fashion. In addition, the functional form chosen is difficult to justify. In contrast, a nonparametric regression model makes few assumptions about the functional form allowing greater flexibility. The two most popular methods for carrying out nonparametric regression are spline based and kernel based nonparametric regression. See Eubank (1988) for discussion about these and other smoothing methods.

In this project, you will study the spline based methods. By developing a Bayesian model and utilizing Markov chain Monte Carlo method to select the appropriate knots of the spline, the final spline nonparametric regression will be estimated using ordinary least square methods.

Eubank, R.L. (1988), Spline smoothing and nonparametric regression (Marcel Dekker, New York).

**Topic:** Statistical analysis of Lotto data

**Supervisor:** Dr Boris Choy

**Contact:** Boris.Choy@uts.edu.au

**Description:** This project investigates the NSW Lotto data using statistical techniques and aims to discover some interesting features from the data. Descriptive statistics will be provided and various statistical models will be used to study the data in deep.

**Topic:** Error Bounds for Lattice Rules for Multiple Quadrature

**Supervisor:** Dr T. Langtry

**Contact:** Tim.Langtry@uts.edu.au

**Description:** Lattice rules are quasi-Monte Carlo rules for the approximation of integrals over the unit cube in  $\mathbb{R}_s$ . The aim of this work is to generalise to the case of higher-rank lattice rules the existing bounds, in terms of Diophantine approximations, on the error of rank 1 simple lattice rules.

**Topic:** The Riemann Zeta function and the distribution of prime numbers.

**Supervisor:** Dr Mark Craddock

**Contact:** mark.craddock@uts.edu.au

**Description:** The prime number theorem describes the approximate, asymptotic distribution of the prime numbers. In this project we will go over the proof of the prime number theorem and look at the connections between the prime numbers and the Riemann Hypothesis. This project is suited to a Mathematics student who enjoyed Advanced Calculus and Analysis 1.

**Topic:** Harmonic Analysis on semisimple and nilpotent Lie groups/ Representation theory of Lie symmetry groups and symmetry methods for PDEs

**Supervisor:** Dr Mark Craddock

**Contact:** mark.craddock@uts.edu.au

**Description:** Lie groups are important in mathematical analysis, physics and geometry because they serve to describe the symmetry of structures. Sophus Lie introduced them in order to study symmetries of differential equations. In this project we will look at Lie symmetry analysis for differential equations and related problems in representation theory.