Berry-Esseen Type Bound for High Dimensional Approximation of Wilks' Lambda Distribution

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Let $\Lambda = |S_e|/|S_e + S_h|$, where S_h and S_e are independent random matrices with Wishart distributions $W_p(q, I)$ and $W_p(n, I)$, respectively. Then Λ has Wilks' lambda distribution $\Lambda_{p,q,n}$ which appears as the distribution of various multivariate likelihood ratio tests. In the case of large n and fixed dimension p the distribution of $T = -n \log \Lambda$ can be approximated by chi-square distribution with $p\,q$ degrees of freedom. Moreover, it is possible to derive the computable Berry-Esseen type error bounds of the approximation (see Fujikoshi and Ulyanov (2005)).

In the talk we consider a case when p is large. The limiting distribution of

$$T_{LR} = \frac{\sqrt{p}}{a\sqrt{q}} \{-\log \Lambda - q \log(1 + q)\}$$

is the standard normal distribution (see e.g. Tonda and Fujikoshi (2004)) under condition that $p/n \to c \in (0, 1)$, where $r = p/m, \quad m = n-p+q, \quad a = \sqrt{2}r/\sqrt{1 + r}$.

We derive a Berry-Esseen type bound for the normal approximation of T_{LR}, that is, we get a computable bound $D = D(n, p)$ such that

$$\sup_x |\mathrm{P}(T_{LR} \leq x) - \Phi(x)| \leq D,$$

for all values of n and p. The bound D is of order $O((1/n + 1/p)^{1/2})$.

The talk is based on recent joint results with Y.Fujikoshi and H.Wakaki from Hiroshima University, Japan.

References

[1] Fujikoshi, Y. and Ulyanov, V.V. (2005). On accuracy of asymptotic expansion for Wilks' lambda distribution. To appear in J.Multivar.Analysis.

[2] Tonda, T. and Fujikoshi, Y. (2004). Asymptotic expansion of the null distribution of LR statistic for multivariate linear hypothesis when the dimension is large. *Comm. Statist.-Theory Meth.* **33**, 1205-1220.